Derived decompositions of abelian categories

Hongxing Chen

Capital Normal University, Beijing

Derived decompositions are introduced to describe a class of semi-orthogonal decompositions of the bounded derived categories of abelian categories in terms of subcategories of the given abelian categories. A concise criterion is presented for an arbitrary abelian category with enough projectives and injectives to have such a derived decomposition. By this criterion, derived decompositions of module categories are constructed from localizing subcategories, commutative noetherian rings and homological ring epimorphisms. Particularly, we show that a commutative noetherian ring of Krull dimension at most one has a derived stratification with abelian simple factors. In general, the bounded derived module category of an indecomposable commutative ring does not have non-trivial stratification by bounded derived module categories of rings. Compared with this phenomenon, the notion of derived decompositions may be of interest for stratifying bounded derived categories of rings by bounded derived categories of abelian categories. Moreover, derived decompositions also supply a construction of recollements and triangle equivalences of bounded derived categories of abelian categories. This is a joint work with Changchang Xi.