Unbounded derived categories and the finitistic dimension conjecture

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I will consider the question of whether the derived category of a ring (unbounded, with no finiteness condition on modules) is generated, as a triangulated category with arbitrary coproducts, by the injective modules, in which case we say that "injectives generate". In a talk in 2001, Bernhard Keller pointed out that if injectives generate for a finite dimensional algebra then the algebra satisfies several homological conjectures, including the generalized Nakayama conjecture, and asked whether there is a connection with the finitistic dimension conjecture. I will prove that if injectives generate for a finite dimensional algebra then the algebra satisfies the finitistic dimension conjecture.

There are rings for which injectives do not generate, but I know of no examples of finite dimensional algebras for which they do not.

References

[1] Bernhard Keller, *Unbounded derived categories and homological conjectures*, Talk at summer school on "Homological conjectures for finite dimensional algebras", Nordfjordeid, 2001.

[2] Jeremy Rickard, *Unbounded derived categories and the finitistic dimension conjecture*, arXiv:1804.09801, 2018.