The Gerstenhaber bracket in Hochschild cohomology

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Homological methods provide important information about the structure of associative algebras, revealing sometimes hidden connections amongst them. This talk will be about an invariant preserved by derived equivalences: the Gerstenhaber bracket in Hochschild cohomology of unital associative algebras over a field. There has been a significant amount of effort expended by many authors in order to study this structure, specially in recent times.

In particular, the Gerstenhaber bracket provides the first Hochschild cohomology space of a Lie algebra structure. The computation of Hochschild cohomology requires a resolution of the algebra considered as a bimodule over itself. Of course, there is always a canonical resolution available, the bar resolution, very useful from a theoretical point of view, but not very satisfactory in practice: the complexity of this resolution rarely allows explicit calculations to be carried out. The use of alternative resolutions is not well adapted to the computation of the Gerstenhaber bracket. However, some results by Witherspoon–Negron, Volkov and Suárez Álvarez provide useful tools to solve this problem. I will illustrate how, using these methods, it is possible to describe the first Hochschild cohomology spaces of some families of algebras as Lie algebras, and relate this to the structure of the algebras.